APPENDIX A

DICTIONARY OF TERMS USED IN MATHEMATICAL LITERATURE

The following little dictionary lists terms which are used in mathematics lectures, text-books and exam papers. They are terms which have to do with the structure and methodology of the subject rather than any specific content.

Algorithm A computational procedure.

As = since

Assume Introduces an assumption which remains in force for the current section of the argument.

Axiom One of a set of assumptions that underlie a whole branch of Mathematics. The entire theory is built on that foundation. These were once considered to be "self-evident truths" such as the axioms of Euclidean Geometry but they are in fact simply part of the definition of some fundamental concept.

Euclid's axioms of Euclidean Geometry were once considered to be physical truths, but mathematicians now regard them simply as part of the definition of the Euclidean Space. Whether the geometry they define corresponds to physical space is a matter for physicists not mathematicians.

Other well-known axiom systems are the group axioms, and the axioms for a vector space. They are simply parts of the definition of a group or a vector space.

Because = since

Choose This is essentially a "let" defining an arbitrary element of a set. *It is important to ensure that the set is non-empty!*

Clearly, obviously Introduces a statement which is claimed to be self-evident. It is designed to prevent the reader from overlooking what is right in front of his or her nose and searching for some deep and far off reason. Sometimes this tag is used where the reasons are far from transparent but where the author could not be bothered to provide the details.

Conjecture A statement which has not been proved but which is felt to be true by the author, and in many cases something which is believed to be true by the mathematical community based on their experience and intuition. There are many famous conjectures whose proofs are actively sought after.

Consider Introduces an item without yet making a statement about it.

Contradiction The set of assumptions currently in force are a logical impossibility. One of them is therefore false (relative to the others).

Contrapositive The contrapositive of a theorem of the form "if p then q" is "if not q then not p". These are logically equivalent forms and frequently a theorem will be proved by proving its contrapositive form. Do not confuse this with the following.

Converse The converse of a theorem of the form "if p then q" is "if q then p". *For many theorems the converse is false.* If the converse is true it must be proved separately. It does not follow from the theorem itself.

Corollary A supplementary theorem which follows from the previous theorem (or corollary) with a little more work.

Counter example An example which proves that a statement of the form "for all x ..." is false by providing one x where it fails.

Definition A statement which introduces a concept or a piece of notation or terminology and gives its precise meaning.

Draw This asks for a reasonably accurate diagram, drawn to scale.

Example A particular case of a theorem (or definition) which serves to illustrate it by showing it operating in a specific case. Unlike a counter example it doesn't prove anything.

Exercise A fairly routine task which the reader is invited to undertake to reinforce his or her understanding.

Explain why, give reasons Give a brief outline of a proof (omit details — just present the key ideas).

Find ..., what is ... Again brief reasons are expected.

Graph This asks for a reasonably accurate plot, drawn to scale.

Hence = therefore

Hence, or otherwise, show that The examiner will accept any proof but gives the clue that you will find one arising from the previous statement.

Hence show that ... requires a proof which makes use of the previous statement.

If makes a temporary assumption which remains in force only until the end of the current statement.

If and only if "p if and only if q" means "p $\leftrightarrow$ q"

Iff = if and only if

Is ... ?, determine whether ... Although this appears to invite a yes/no answer the intention is usually that you give supporting reasons.

It can be easily (readily) shown that introduces a statement whose proof is omitted because it is routine. The reader is expected to be able to supply the details.

It can be (has been) shown that introduces a statement whose proof is omitted because it is too difficult, too long or too tedious.

It follows that = therefore

Lemma a theorem which has little independent interest but which is a self- contained result which will be used in the following theorem.

Let defines a symbol. It can be a constant (let $x = \pi^2 + 1$), or it can define a substitution (let $u = x^2 + 1$), or it can define something that uniquely has a certain property (let n be the smallest positive prime p such that $2^p - p^2 > 1000$. Or it can be used to introduce an arbitrary variable (let $x \in S$). Every symbol (other than those which have conventional meanings such as π) must be declared. Sometimes "let" is wrongly used instead of "suppose".

Necessary and sufficient condition "p is a necessary and sufficient condition for q" means that "$p \leftrightarrow q$".

Necessary condition "p is a necessary condition for q" means "$q \rightarrow p$".

Now This indicates a fresh start. The previous statement is put on hold while another piece of the argument is assembled.

Problem Usually this is a less routine task which requires not only understanding but also some creativity (original thinking).

Proof A chain of reasoning which validates a theorem, lemma or corollary.

Prove that Create a proof for the statement.

Q.E.D. Quad erat demonstratum = that which was to be proved. This marks the end of an argument. Sometimes books use special symbols such as ☐ or # for the same purpose.

R.T.P. Required to prove. This is a signpost which identifies the current goal. This goal should never be stated without some tag to indicate that it has not yet been proven.

Show that = prove that

Since Introduces a reason

Sketch Asks for a rough drawing, not necessarily to scale, which nevertheless displays the important qualitative features (eg. turning points).

So = therefore

Sufficient condition "p is a sufficient condition for q" means "$p \rightarrow q$".
Suppose = assume

Theorem A statement for which a proof exists.

Therefore Introduces a statement which follows logically from the previous one (perhaps in conjunction with earlier ones).

Well-defined If a definition is expressed in terms of a non-unique representation of something there is the potential problem that different representations lead to different definitions. A function (operation etc.) is well defined if different representations give rise to the same definition. For example the function $f:\mathbf{Q}\to\mathbf{Q}$ defined by $f(m/n) = (m+n)/mn$ is not well defined since, although $1/2 = 3/6$, $f(1/2) = 3/2$ while $f(3/6) = 9/18 = 1/2$. The operations of addition and multiplication of rational numbers are defined in terms of numerators and denominators. However it can be checked that these operations are well-defined, something you've taken for granted all your life, probably without giving it a moment's thought.

w.l.o.g. = without loss of generality. This is where, for convenience, a particular case is considered which trivially leads to the general case. For example given two distinct integers i, j, with identical assumptions on each, we may suppose w.l.o.g. that $i < j$ because if $i > j$ we simply interchange the symbols.

APPENDIX B

PATTERNS OF PROOF

$\neg p$	$p \vee q$	$p \wedge q$	$p \to q$	$p \to \neg q$	$\neg p \to \neg q$	$p \leftrightarrow q$
Suppose p .... Contrad'n! Hence p	Suppose p is false .... Hence q	.... Hence p .... Hence q	Suppose p .... Hence q	Suppose p Suppose q .... Contrad'n!	Suppose q .... Hence p	Suppose p .... Hence q Now suppose q .... Hence p

$p \vee q \to r$	$p \wedge q \to r$	$p \to q \vee r$	$\exists x \in S\ [Px]$	$\forall x \in S\ [Px]$	$\forall x \in S\ [Px \to Qx]$
Case I Suppose p .... Hence r Case II Suppose q .... Hence r	Suppose p Suppose q .... Hence r	Suppose p Suppose q is false .... Hence r	Let x = ... Hence Px	Let $x \in S$ .... Hence Px	Let $x \in S$ Suppose Px .... Hence Qx

PROOF BY INDUCTION

ALL SIX STEPS MUST BE PRESENT	**EXAMPLE:** Prove that $T_n = n(n-3)$is even for all $n \geq 3$.
(1) **CHECK** the first value	If n = 3, $T_n = 3 \times 0 = 0$ which is even. Hence the statement holds for n = 3.
(2) **SUPPOSE** result is true for n	Suppose the statement holds for n i.e. suppose T_n is even
(3) **CONSIDER** the n +1 case	$T_{n+1} = (n+1)(n+1-3) = (n+1)(n-2) = n^2 - n - 2$
(4) **RELATE** it to the n case	$= (n^2 - 3n) + 2n - 2 = T_n + 2(n-1)$
(5) **USE** induction hypothesis to prove the n+1 case	Since T_n is even (by assumption)and $2(n-1)$ is even, then T_{n+1} is even.
(6) **CONCLUDE** proof by appealing to the induction principle	Hence by induction, T_n is even for all $n \geq 3$.