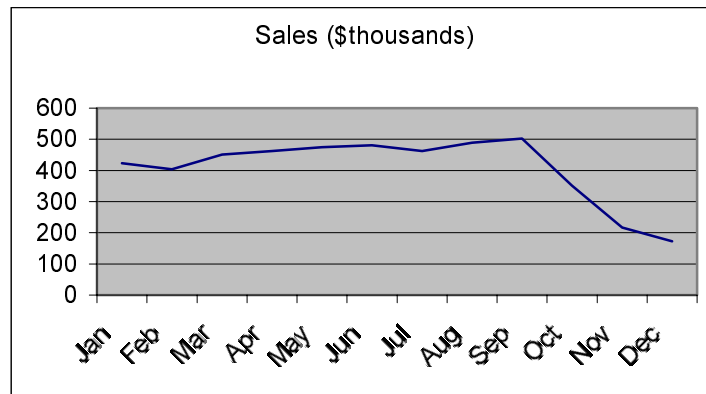


1. GRAPHS AND THE STORIES THEY TELL

1.1 The Height of Points on a Graph

We're all very familiar with graphs as a way of depicting information. You always have two quantities and the graph shows how one is related to the other. For example the weekly sales of a small company can be plotted against time.



It's possible to read off the monthly sales figures from the graph (at least approximately). For example in February the sales were about \$400,000. But if we just want these figures it would be better to have a table of values.

A graph is particularly good at highlighting trends. Without getting bogged down in the actual figures we can instantly see there was steady growth up until September. Something drastic happened in September and the bottom fell out of the market.

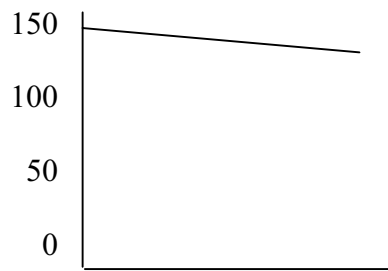
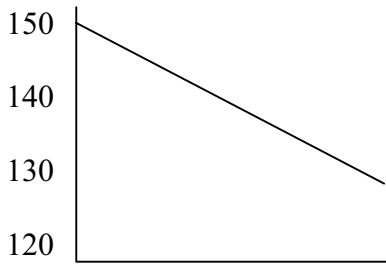
This graph isn't very smooth. This is because there are many factors that affect the level of sales. There's no way we could describe it by a simple formula. But there are some instances where there's a simple underlying law that results in a simple formula and a smooth graph. For example if we plot the temperature of a cup of tea against time we would get a graph like this.



The main feature of a graph is the height of the points on it. This reflects the size of the quantity being plotted. If sales are up the points on the graph are higher. If they drop the points are lower. We can read off these values from the scale. But more importantly we can see visually the rises and falls much more easily from a graph than we could from a corresponding table of values.

So the graph gives us an instant picture of what is happening. We don't need to look up values and decide whether one value is greater or less than another. The graph shows us the relative values in an instant.

Now there's one trick that is often used to exaggerate the relative differences between the values of some quantity. This is the use of a "truncated scale". Consider the following two graphs.



If you read off the values you'll see that they both display the same figures. The emotional impact of the first graph is very different to that of the second. If the graphs represent sales figures the impression given by the first graph is that the company is in serious trouble while the second graph gives the more correct impression that sales are down a bit. The eye tends to take in the proportions of the heights of the points on the graph and the first graph gives the wrong impression that sales have dropped to one third of what they were.

Truncated scales are often unavoidable. To continue the scale right down to zero may be impractical and waste a lot of space. But you should be aware of the distortions that can result. The effect of truncating the scale is to change the slope of the graph – make it look a lot steeper than it really is. And the slope of a graph is a second feature of a graph which, used correctly, can convey very useful information.

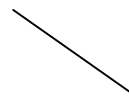
1.2 The Slope of a Graph at a Point

Just as important as the height of the points on a graph is whether the graph is going up or down at a particular point and how steep the graph is at that point. These two features are incorporated into something called the **slope** of the graph. For a sales graph, going up is good while going down is bad. If a graph is going up at some point this reflects the fact that the variable (in this case sales) is increasing. If it is going down the quantity is decreasing. For the cooling graph the curve is going downhill all the way. But it gradually levels out as it gets closer and closer to room temperature.

We call an upwards slope (going from left to right) a positive slope. A downwards slope is called negative. For a sales graph, a positive slope is good and a negative slope is bad. (But if we were plotting road fatalities we would be happy to see negative slope.) The slope of a cooling graph is negative all the way.



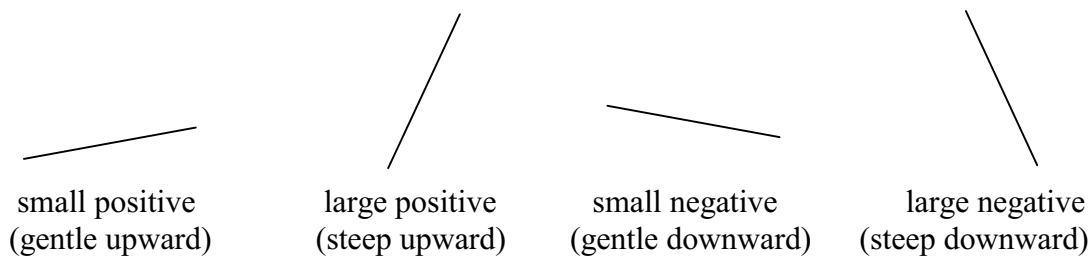
positive slope



negative slope

The direction of the slope tells us one thing, but the actual steepness tells us more. The sudden sharp drop in sales as we pass through September is telling us a different story to that which would be told by a gentle negative (downward) slope. The size of a slope is the steepness and we'll soon learn how to calculate or estimate it. For now we'll just say that steep slopes have large values and gentle slopes have small values. But we have to combine these notions of large and small values with positive and negative.

We can consider therefore four basic types of slope.



Of course an upward slope for me is a downward slope for you if I meet you going in the opposite direction. But whenever we describe a slope mathematically we always do it from the perspective of someone moving from left to right.

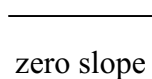
Now if these slopes were slopes along a path we'd respond very differently to each of them physically! The most pleasant slope would be the gentle downward (small negative) slope. The gentle upward slope (small positive) slope requires a bit more effort but nothing like what's needed for the steeper slopes. You'd need to be very fit to negotiate either of these large slopes. And the large upward slope would require rather different climbing techniques to the large downward one.

But some slopes are steeper and gentler than others so it doesn't really make sense to simply classify slopes into just these four categories. Instead we quantify slope and express it numerically.

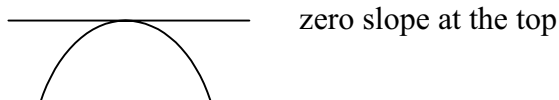
The size of a slope, ignoring whether it's positive or negative (up or down) is the horizontal distance divided by the vertical distance.

The gentle upward slope above looks as though it might be a 1 in 4 slope (1 unit up while going 4 units forward). We describe this slope as a slope of $\frac{1}{4}$. The gentle downward slope illustrated above is also about 1 in 4, but being a downward slope, when moving from left to right, it is negative. So we give it the value $-\frac{1}{4}$. The steep slopes illustrated above may perhaps be 4 and -4 respectively.

Of course sometimes a path might be running level, neither up nor down. This would be quantified as a slope of 0.



But the curve doesn't have to be a straight line for this to occur. At the top of a hill, at the point where you stop climbing and start descending, the path is momentarily horizontal, or has slope zero.



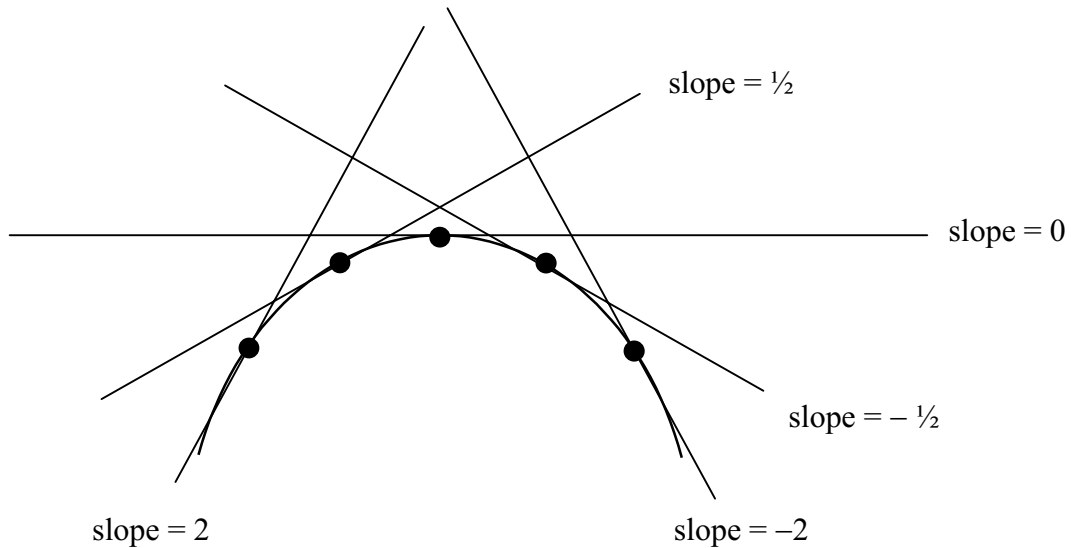
Notice that we drew this horizontal line, balancing on the curve, to highlight the fact that the curve is momentarily level at the top. Such a line is called a **tangent**. It touches the curve at just one point and has the same slope as the curve does at that point.

A smooth curve has a tangent at every point. And as the point moves along the curve the tangent changes its slope. We define the **slope** of a curve at a given point to be the slope of the tangent at that point.

Definition: The **slope** of a straight line is $\frac{\text{Rise}}{\text{Run}}$, that is, the vertical distance divided by horizontal distance. These distance refer to any two points on the line with the horizontal distance measured positive and the vertical distance measured positive if it is upwards (going from left to right) and negative if it is downwards. The slope of a horizontal line is zero. A vertical line has no slope (sometimes we say that it has infinite slope).

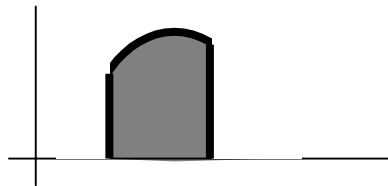
Definition: The slope of a curve at a point P is the slope of the tangent at that point.

Example 1:

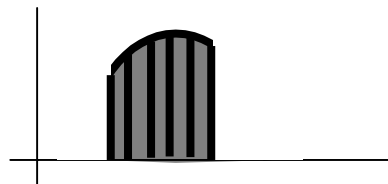


1.3 The Area Under a Graph

The third important feature of a graph is the **area** underneath the graph between two vertical lines. This is the area above the horizontal axis and enclosed by that axis, the two vertical lines, and the curve.

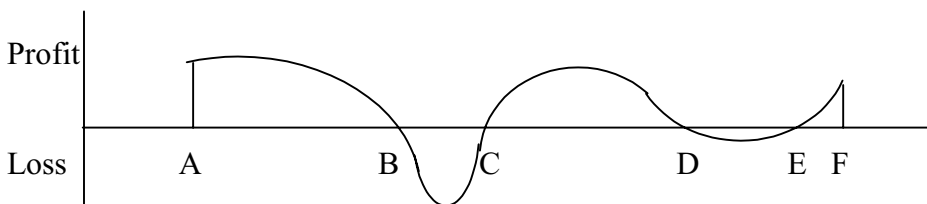


Such an area can be approximated by a whole lot of thin strips with constant width. The area under the curve will be the sum of the areas of these strips.



Now each of these strips approximates to a rectangle. If this is a graph of weekly sales and the width of the strips corresponds to one week, the area of each strip would represent one week's sales. The total of all the strips would therefore represent the total sales over the whole period. If there are 52 such strips and the two outside vertical lines corresponds to the beginning and end of the year, the area under the curve would represent the total sales for the year.

If the graph represents the weekly profits, rather than weekly sales, the curve could drop below the axis. A negative profit would be interpreted as a loss. During those weeks when the curve is below the axis the total area represents the accumulated *loss*. This would be offset against the profit gained during those weeks when the curve is running above the axis. In order that losses get treated as negative profits, to be subtracted away from any actual profits, we treat areas down from the axis as negative.



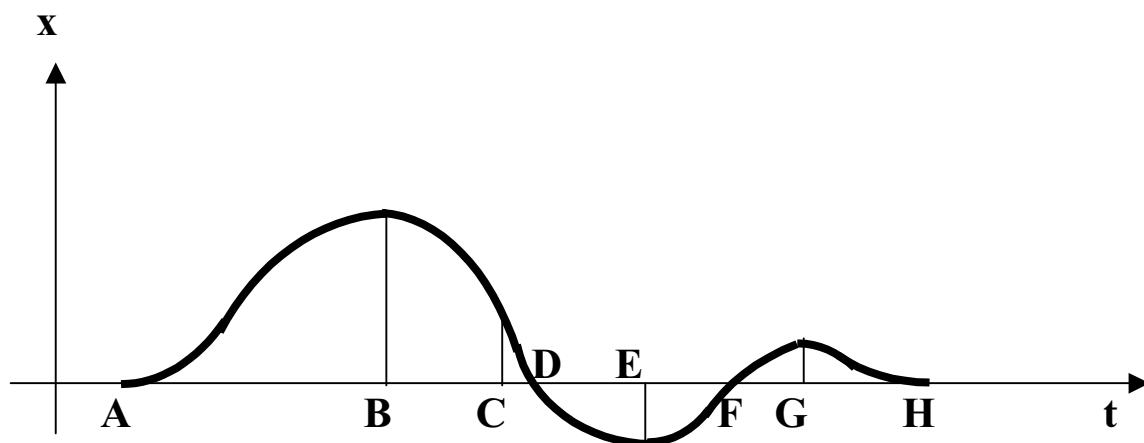
This graph shows a period of profit from A to B followed by a period of loss from B to C. The total profit accumulated from A to B is represented by the area under the graph from A to B. The following loss is represented by the area above the curve from B to C. While it isn't easy to quantify these profits and losses (especially as we haven't provided a scale) one can easily see that the area below the graph from A to B is rather more than the area above the curve from B to C. This means that the company should still be solvent at time C. The net profit during the period from A to C will be the area below the graph from A to B *minus* the area above the curve from B to C. This will be a net profit, rather than a loss.

Over the whole period from A to F the accumulated profit will be represented by the entire area between the graph and the axis **provided we treat areas below the axis as negative**.

1.4 The Stories Behind a Graph

Graphs tell stories, and an important skill is to be able to read the story behind the graph. Before we start to become too technical about graphs we'll get some practice in interpreting graphs. This will involve using a little imagination. But as you explore the stories behind the following graphs be on the lookout for the three fundamental ingredients of a graph: height, slope and area.

Have a good look at the following graph. Certain points have been given names so that we can refer to them easily. The graph represents a certain variable, called x , plotted against another variable called t . We haven't said what these variables are. They will vary from story to story. Also we haven't specified the scale. This is because we don't want to get bogged down by the actual values of the variables. We want to focus on the qualitative features of the graph.



Story #1: Cross-section of a Mountain

Suppose the graph represents the cross-section of some countryside, viewed towards the east. Both variables here represent distances. The variable x represents a distance along the ground in the north-south direction while the variable x represents the height of the land at that point. It is likely that the same scale is not used on both axes because the distance from A to H might be 20 or 30 kilometres while the total height of the mountain might be only one or two kilometres. But, as we said, we are not bothering about the scale.

Question 1: Where is the highest peak? Clearly the answer is at the point B.

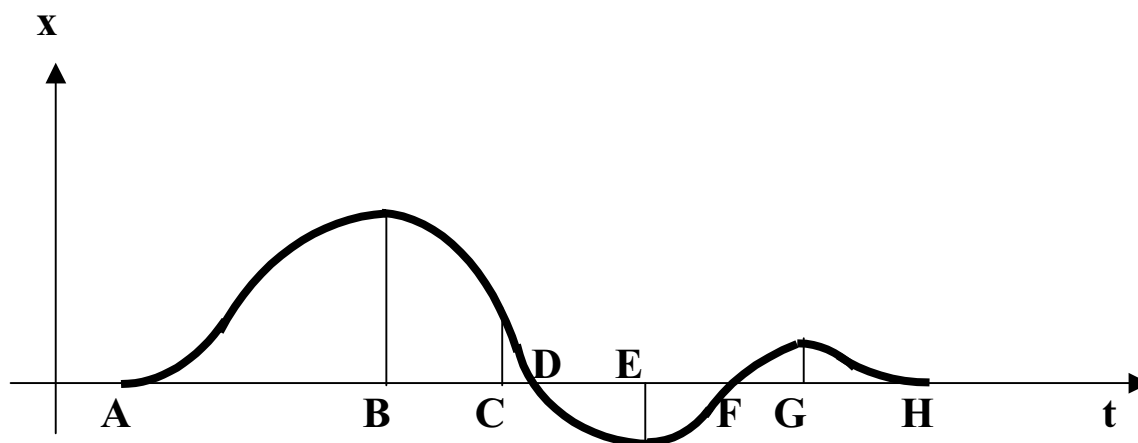
Question 2: Where is the mountain steepest? There's a steep section between A and B but probably, of all the labelled points, C seems to be the steepest.

Question 3: A river flows through this landscape, from north to south. Where is the bottom of the river? It would have to be at E. For land to hold a river it must slope up on either side. Looking from left to right the slope must be negative down to the river bed and positive up the other side. At the lowest point, E, the slope is neither positive or negative — it is zero.

Question 4: Where is the best place for a farm? Here is a somewhat subjective question so any answer cannot be defended with mathematical certainty. But we can rule out certain points. E would be a stupid place if we are assuming that it is the bed of a river. B might be too exposed. You could make a case for G. It's reasonably flat and not too far from water. You might have come up with a different answer. That's OK as long as you can give a plausible argument to defend it. The goal of this exercise is to get you to be able to *interpret* the graph.

Notice that the features of the graph on which you based your answers were height and slope. Now for a second interpretation using the same graph.

Story #2: Side View of a Roller Coaster



Here the graph is being interpreted as the side view of a roller-coaster. As in the first story, t represents horizontal distance and x represents vertical distance.

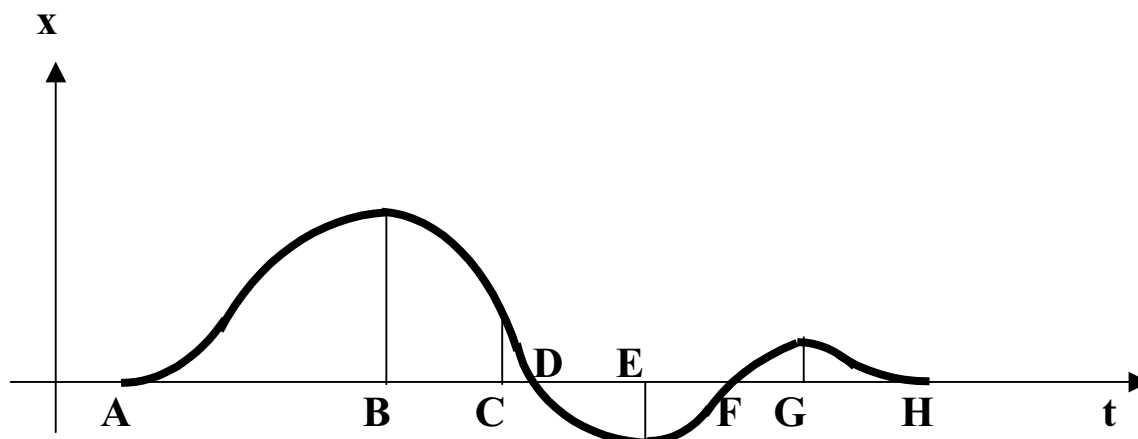
Question 1: Where would you get the best view? Probably your answer would be at B, the highest point. Notice that although this point is characterised by height — the highest point — it is also one of the “flat” points, where momentarily the graph is neither going up or down.

Question 2: Where is the safest place to stop the roller-coaster car without putting on the brakes. This could be any of the points where the curve is flat (zero slope): A, B, E, G, H. But certain of these points would be a little dangerous. You could park the roller-coaster car precariously at B but a puff of wind might push it over the edge. Probably the safest place is at E.

Question 3: At what point might the roller-coaster car be going fastest? There are a lot of assumptions to be made here but you could argue that while it is going down-hill it picking up speed so that it might be going fastest at E. This is one of the minimum points (where the slope is zero). It might be going fast at the minimum at H but it hasn't been going down-hill for as long as when it hits B. Again it doesn't matter if you disagree with this answer as long as you can come up with good reasons.

Question 4: What point is the scariest? This is another one of those subjective questions. It all depends on what frightens you. If you are afraid of heights then B might be the scariest — it's the highest. If you're scared of speed then perhaps E is the scariest. If you are afraid of falling out of the car then the steepest point, C, would be the scariest. Take your pick!

Story #3: Position of a Cyclist



Both of the previous stories have had the graph give a literal picture of the situation and the interpretation has been relatively easy. But in this next story the horizontal axis represents time. The vertical axis represents distance along a north-south straight road. The graph gives the position of a cyclist as he rides up and down this perfectly straight, perfectly flat road. The graph curves around but not the cyclist.

The horizontal axis represents his house. Distances above this axis represent the cyclist being north of home and distances below this axis represent distances south. He leaves home at time A, riding north. Then at time B he slows down and stops in order to turn around. Between B and E he is going south, and at some point he passes his house. At time E he turns north again, and so on.

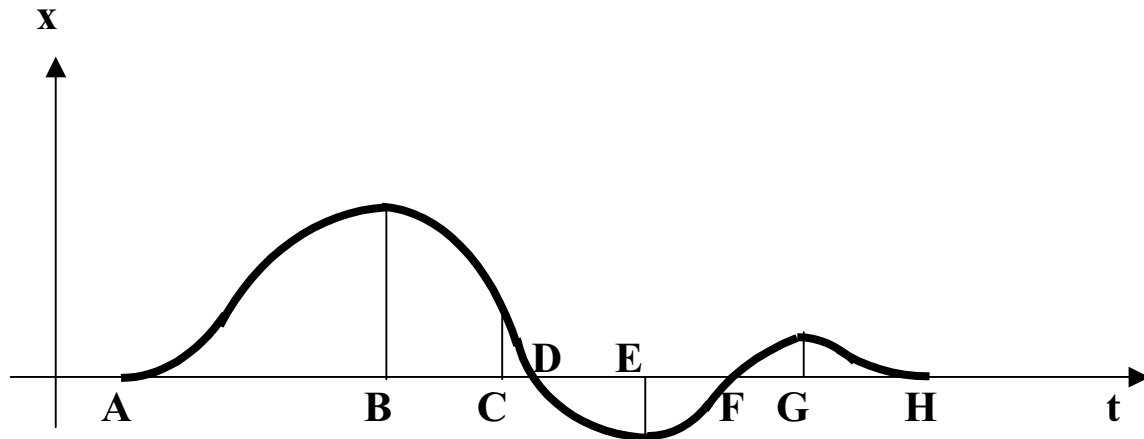
Question 1: He starts outside his house at time A and ends up outside his house again at time H. How many times does he ride right past his house? The answer is 2. He is riding past his house at times D and F.

Question 2: At what point is he riding fastest? The slope on this graph measures how fast he is travelling. The steepest point on the graph is at C and this represents the point where he is covering the most distance in each time interval.

Question 3: Does he spend more time north of home or south of home? Although we don't have a scale it is clear that he has spent more time north of home. He is north of home between time A and D and again between F and H. This clearly represents much more than half of the total time.

Question 4: At what point in time did he realise that it was dinner time? One can't be certain but a good guess would be at time G when he stopped and returned home to stay.

Story #4: Speed of a Train



In this story a train is moving along a straight, perfectly flat, piece of track going north-south. The horizontal axis represents time while the vertical axis represents the speed of the train in a northerly direction. The higher the graph, the faster the train is going. The horizontal axis represents a speed of zero, that is, the fact that the train is stationary. When the graph goes below the axis the train is going south. A speed of -60 kph is interpreted as a speed of 60 kph southwards.

Because we have no scale on the graph we can't say how fast the train is going at any given moment. But this allows us to focus on the qualitative aspects of the scenario.

The train is stationary at time A but immediately starts to move north. It goes faster and faster until at time B it is going fairly fast. (We can't say exactly how fast, but we can say that it is going faster than at any other time during the whole period from A to H.) From B to D the train is slowing down, but it is still travelling north. The cyclist, whose story was given by the same graph turned around at time B and started heading south. The difference has nothing to do with the fact that he is a cyclist while this is a train. It is simply that here we're plotting speed not position.

But at time D the train stops momentarily. Since the graph then dips below the axis we interpret this as the train going south. So at time D the train stopped and started reversing back down the line. Again it picks up speed, although it goes more slowly than when it was going forwards. At time E it starts slowing down again until at time F it has again stopped in order to go forwards again. It travels a certain distance north until again it slows down and stops at time H.

Question 1: When is it going fastest?

Clearly at time B. The fastest it ever goes backwards is clearly at time E but since the distance to the axis is less than at B the speed it attains is less. Also at G it is going faster than just before or just after G, but clearly this isn't as fast as at time B. Notice that all three of these moments, B, E and G correspond to places where the slope of the graph is zero.

Question 2: When does it change direction?

The moments are D and F. Notice that at these times the graph cuts the axis, meaning that the train is momentarily stationary. It has to stop in order to change direction. The train is stopped at times A and H as well but because we don't know what happened before A or after H we can't say whether the train changes direction at these times.

Question 3: When is the train furthest from the start?

The answer is D. But follow this analysis carefully. From A to D the train is travelling north so at time D it is further north than at any time before. But it goes back down the line a certain distance and then back north again. If, by time H it hasn't returned to where it was at time D then it will be furthest from the start at time D. But if, after backing down the line, it goes north and passes the point where it was at time D then the time when it is furthest from the start is at

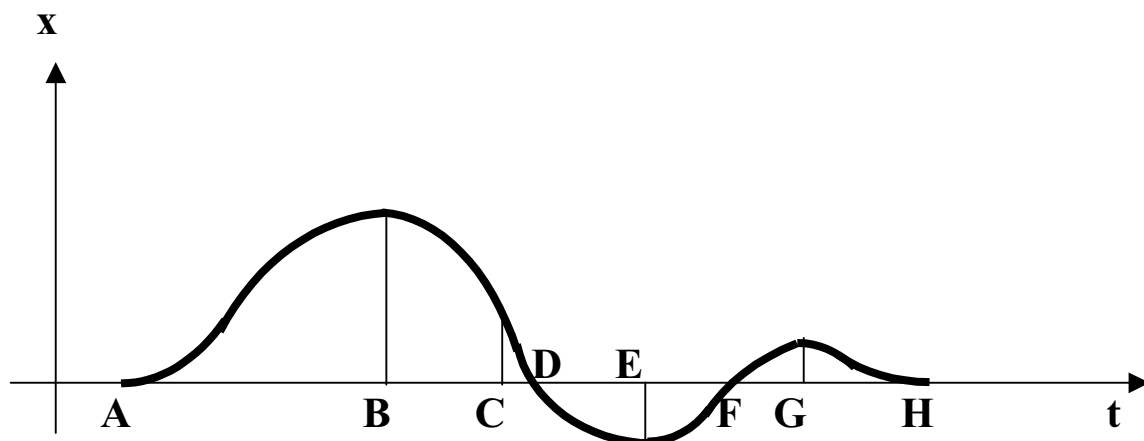
time H. How can we decide between these two possibilities since we have no scale on which we can base our calculations?

That's easy. The area under the curve represents the distance travelled. Imagine that the area is divided into strips with the width of each strip representing one minute. We can approximate these strips by rectangles. The height of each rectangle represents the speed at that stage and so the area of a rectangle represents the distance travelled in that particular minute. Adding up all these rectangles gives the total distance travelled north. Areas below correspond to negative distances north — that is, distances south.

Now by approximating the strips by rectangles we may think that the area under the curve only approximately gives the distance travelled. But in fact the speed is changing continuously and if we used much narrower rectangle, each representing one second of time, the strips and the rectangles would be very much closer to each other. It can be shown that the area under the graph *exactly* represents the distance travelled.

The area of the piece below the axis seems to be bigger than the third area, between F and H. This means that when it goes north for the second time it never reaches where it was at time D and so at time D the train was further from the start than at any other time.

Story #5: Weekly Savings



Once again the horizontal axis represents time, over the lifetime of Old Jock. Time A was when he started working and time H was when he died. The vertical axis represents his weekly savings. When the graph goes below the axis this represents negative savings, that is the times when he has to withdraw money from his account in order to live.

Question 1: During what period does he have to dip into his savings?

Clearly this is between time D and time F.

Question 2: When does he have the largest bank balance?

His bank balance is greatest at time D. The amount he saves weekly is really the speed at which his bank balance is growing. Just as in the last story, the area under the graph represents the accumulated savings. Just imagine the area broken up into lots of thin strips, each representing one week. The area of each strip would represent the amount saved in the week so the total area would represent total savings. Where the graph dips below the axis the area gets counted negative and represents the total amount withdrawn over the period.

The area from A to D represents his bank balance at time D. From time D to time F he has to withdraw and his bank balance at time F would be the area from A to D minus the area from D to F. (If we count the area from D to F as a negative area then we would be able to add the two areas.) After time F he is able to save again and so his bank balance increases again. But

amount saved from F to H is clearly not enough to offset the withdrawal from D to F, so he never regains the balance he had at time D.

Question 3: When is his income exactly equal to his expenditure?

The answer refers to the weeks when his net savings is zero. These are at times A, D, F and H.

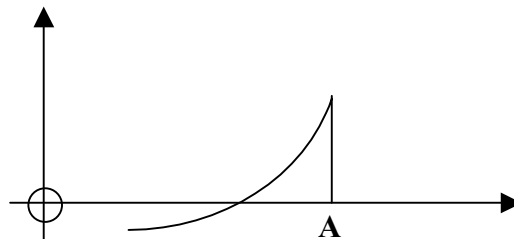
Question 4: When does he get married?

This is a subjective question and different assumptions may lead to different answers. One might conjecture that he got married at around time B as then his expenses clearly started to increase. That’s assuming his wife didn’t go out to work and looked after the children. But it could be that between A and C his expenditure was more or less constant but that he lost his job at time B and his new job paid less. He might then have married at time E and, with the help of his wife’s income, he was able to once again make ends meet. Again the important thing with this exercise is not to so much to be able to come up with the correct answer as to be able to read the graph as one or more possible stories.

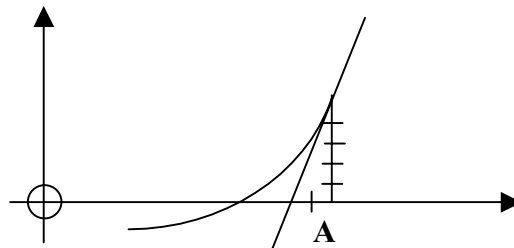
1.5 Estimating Slopes

If we are given a graph and we need to find the slope at a point, the best we can do is to try to draw the tangent at that point and working out its “rise over run”. Because of the difficulty of accurately positioning the tangent this method is subject to very large errors.

Example 2: Estimate the slope to the following graph at the point A.



Solution: Drawing the tangent as accurately as we can we get:



This gives us an estimated slope of $\frac{5}{2} = 2.5$. But, because of the difficulty of drawing the tangent, it could be anywhere between 2 and 3.

Example 3: Find the slope of the graph $y = x^3 - 1$ at $x = 1$.

Solution: We have not yet developed the techniques for finding the slope exactly. But the tangent at the point can be approximated by the slope of the line joining the points where $x = 1$ and where $x = 1.1$, that is, the points $(1, 0)$ and $(1.1, 0.331)$. This slope is $\frac{0.331}{0.1} = 3.31$. A better approximation would be to use the points $(1, 0)$ and $(1.01, 0.030301)$ on the graph. This gives an estimate of 3.0301. (The exact slope, as we shall see later, is 3.)

1.6 Estimating Areas

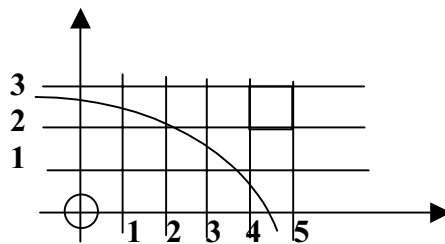
Areas under graphs can be estimated somewhat more accurately than slopes. We simply divide the region into small squares and count the squares. If the sides of the squares are 1 unit then this will give us an estimate of the area. Otherwise we have to multiply the number of squares by the area of each. For example if we had 18 squares, each $\frac{1}{2} \times \frac{1}{2}$, each square would have area $\frac{1}{4}$ and so the total area would be estimated to be $\frac{18}{4} = 4.5$.

But what do we do about squares that are cut by the curve. Part of the square is to be counted and part ignored. We could try to estimate the fraction of the square that is to be included, but a simpler rule is to count the square if more than half of it lies in the region and to ignore it otherwise. This way we are only dealing with a whole number of squares, which makes the counting easier. On average the bits of area you leave out in this way will balance the bits you have counted but shouldn't.

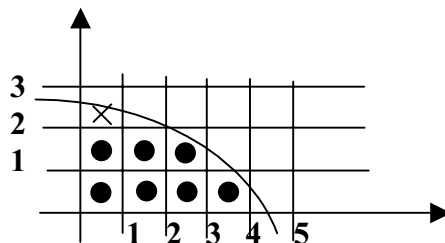
Sometimes, however, you'll get squares where roughly half is inside the region and you'll waste time trying to decide whether or not to include it. In these cases you could count the as half squares.

If you are concerned about the accuracy of your estimate you can use smaller squares. The smaller the squares the less important each one is, and the more accurate will be your estimate.

Example 4: Estimate the area enclosed by the following curve and the axis.

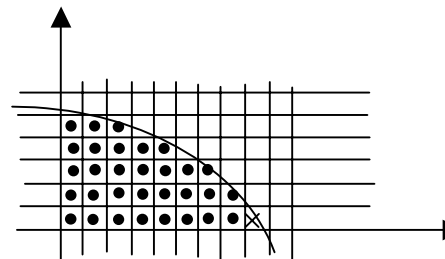


Solution: Mark the squares to be counted as ● and the half squares as ×:



This gives $7\frac{1}{2}$ squares, each with an area of 1 square unit. So our estimate of the area is 7.5.

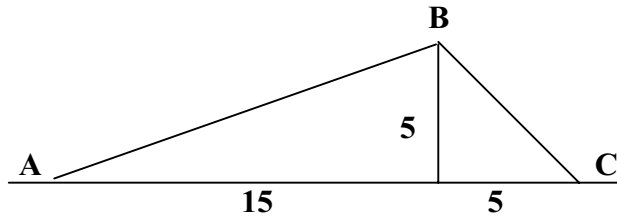
Suppose instead we use $\frac{1}{2} \times \frac{1}{2}$ squares.



Here we have $31\frac{1}{2}$ squares, each with area $\frac{1}{4}$, giving an estimate of $\frac{63}{2} \cdot \frac{1}{4} = \frac{63}{8} = 7.875$. This is bound to be more accurate than the previous 7.5, but probably not accurate to 3 decimal places. It would be more appropriate to express this better estimate as 7.9.

EXERCISES

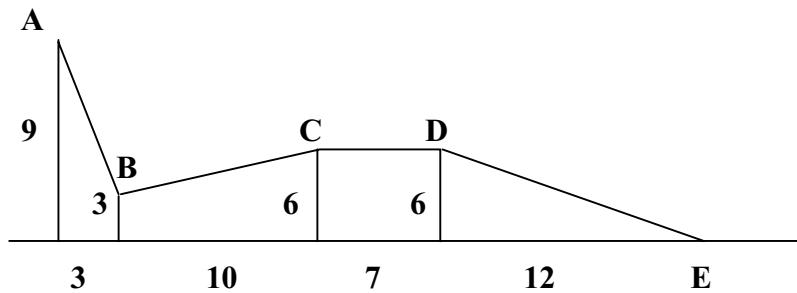
Exercise 1:



The line AC is horizontal. Find the slope of:

- (i) AB ; (ii) BC ; (iii) AC .

Exercise 2:



Find the slope of:

- (i) AB ; (ii) BC ; (iii) CD ; (iv) DE ; (v) AD .

Exercise 3:

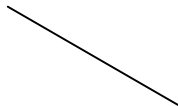
Without measuring distances, judge the following slopes. Choose from the alternatives:

$-2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2.$

(i)



(ii)



(iii)



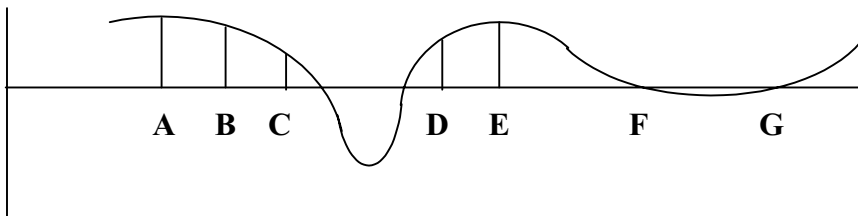
(iv)



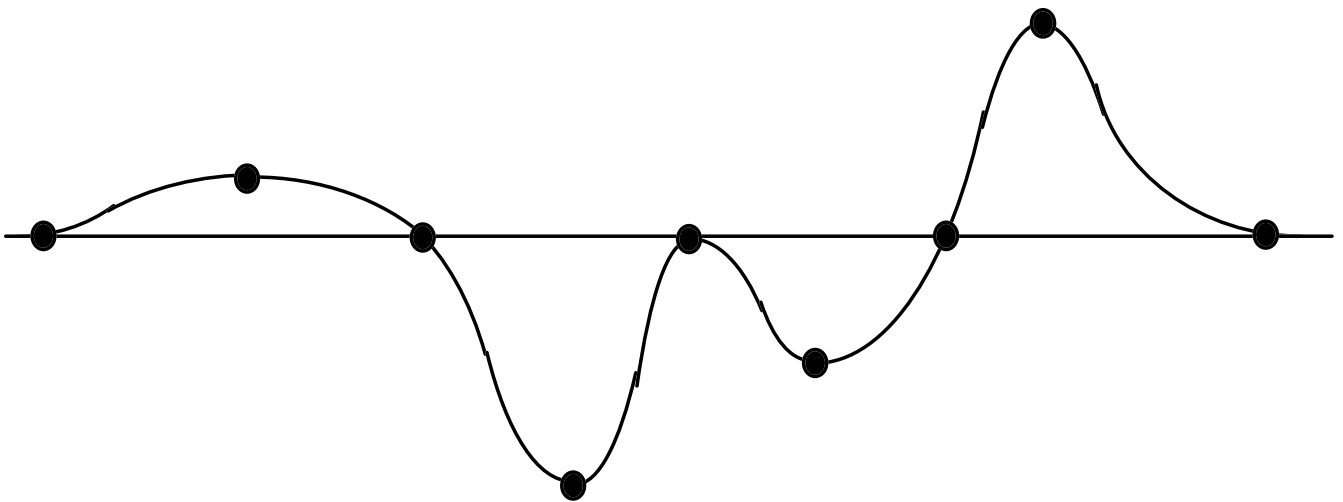
Exercise 4:

Estimate the slope at each of the following points. Choose from the alternatives:

$-2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2.$



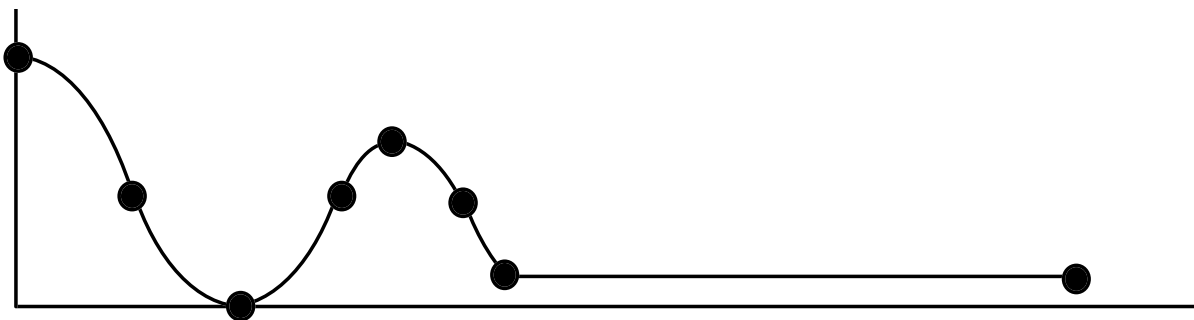
Exercise 5: The following graph represents the Rhine River in the times of the Valkyries. The horizontal line represents the water level in the river.



Where would you expect to find:

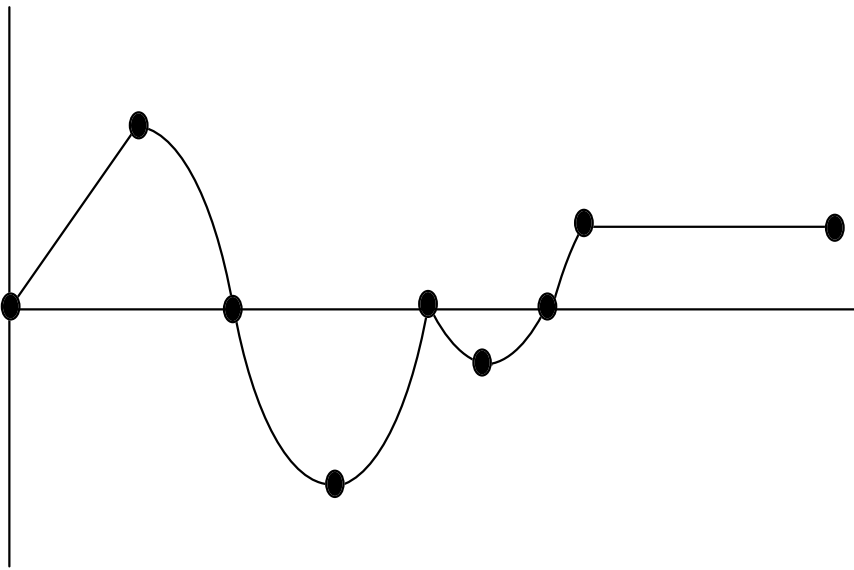
- (a) Valhalla, Wotan's castle?
- (b) The Lorelei who lure sailors onto the rocks.
- (c) The Ring of the Nibelungen, in the deepest part of the Rhine.
- (d) The army of the giant Fafner, assembled ready to make an assault on Valhalla.

Exercise 6: The following graph represents the height above the water level of a bungy jumper who jumps off a bridge. The horizontal level represents water level.



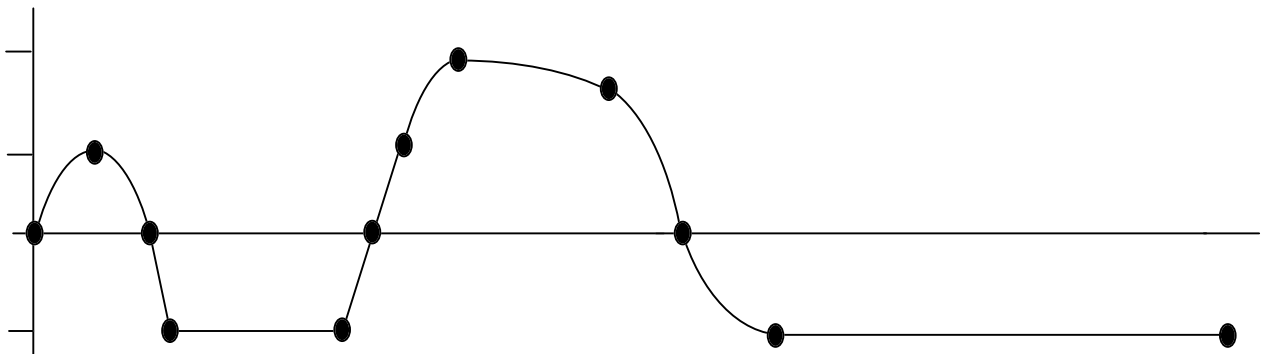
- (a) Where is he falling fastest?
- (b) When does he get his feet wet?
- (c) Does he go under water at any stage?
- (d) What might have happened at G?

Exercise 7: In a laboratory experiment, the following graph represents the angle of a pendulum, measured from vertical, against time, t .



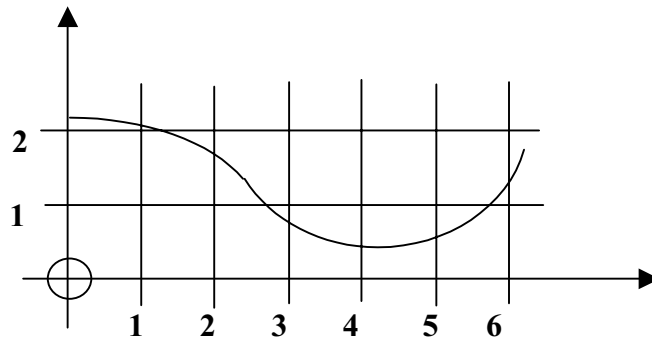
- (a) What is happening between A and B?
- (b) Where is the pendulum travelling fastest to the right?
- (c) What might be happening at E?
- (d) What might be happening between H and I?

Exercise 8: The following graph represents the net inflow/outflow per day, into or out of a dam. The flow rate is plotted against time.

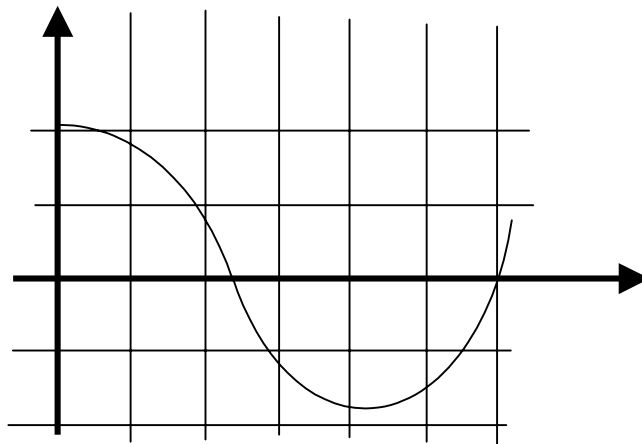


- (a) How much water is pumped from the dam each day? (Assume it's constant and ignore wastage due to evaporation etc.)
- (b) When did it rain?
- (c) When did it rain the hardest? (Ignore any delay there might be in the water reaching the dam.)
- (d) When did the drought begin?
- (e) Is the level of the dam at time F the same, lower or higher than at time A?
- (f) When was the level of the dam the same as at time A?

Exercise 9: Use the Counting Squares Method to estimate the area under this curve from $x = 0$ to $x = 6$. Use 1×1 squares and then repeat with $\frac{1}{2} \times \frac{1}{2}$ squares.



Exercise 10: Use the Counting Squares Method, with $\frac{1}{2} \times \frac{1}{2}$ squares, to estimate the area between this curve and the horizontal axis, between $x = 1$ and $x = 6$. (The squares shown are 1×1 squares.) Remember to count area below the horizontal axis as negative.



Solutions: Chapter 1

Exercise 1:

(i) Slope is vertical rise divided by horizontal run.

$$\text{Slope (AB)} = 5/15 = 1/3$$

(ii) Slope (BC) = $-5/5 = -1$

(iii) Slope (AC) = $0/20 = 0$

Exercise 2:

(i) Slope (AB) = $\frac{3-9}{3} = -\frac{6}{3} = -2$

(ii) Slope (BC) = $\frac{6-3}{10} = \frac{3}{10}$

(iii) Slope (CD) = $\frac{6-6}{7} = 0$

(iv) Slope (DE) = $\frac{0-6}{12} = \frac{-1}{2}$

(v) Slope (AD) = $\frac{6-9}{3+10+7} = \frac{-3}{20}$

Exercise 3:

(i) $\frac{1}{2}$ (ii) -1 (iii) 0 (iv) 2

Exercise 4:

A: 0 B: $-1/2$ C: -1 D: 1

E: 0 F: $-1/2$ G: $\frac{1}{2}$

Exercise 5:

(a) H, on top of the highest hill allowing the castle to be easily defended.

(b) E, where the rocks come up to the surface but cannot be seen from above ground.

(c) D, a minimum on the curve, which represents the deepest part of the river.

(d) Possibly B, because flat solid ground would make a good assembly point, however the army would still have to cross the Rhine River, so perhaps I would be more appropriate for an immediate assault.

Exercise 6:

(a) B, since this is where the graph has the most negative slope.

(b) C, where the curve touches horizontal water level.

(c) No, since his height, represented by the curve, is always above the axis.

(d) Possibly a boat came under the bridge, the man hit the deck, got tangled up and the bungy rope snapped.

Exercise 7:

- (a) The pendulum is being pulled to the left at a constant speed.
- (b) C, the point with the most negative slope. When the pendulum is released at point B it is pulled down by gravity and hence picks up speed, travelling to the right. This continues until it reaches the vertical position (C), at which point it starts to travel upwards and hence lose speed.
- (c) Possibly something is placed in the path of the pendulum, causing it to bounce back to the right.
- (d) While the pendulum is moving to the left the experimenter catches the pendulum and holds it.

Exercise 8:

- (a) 20 Megalitres a day are pumped from the dam. This can be seen from the horizontal areas of the graph, presumably where there is no rain.
- (b) Between points A and D, and then between points E and K.
- (c) H, where the input is a maximum.
- (d) K as there was no inflow for some time.
- (e) Lower, since overall there has been more output than input. This is represented on the graph by a greater area under the curve than above between points A and F.
- (f) The level of the dam would be the same when the area bounded by the curve that is above the axis and below the axis exactly cancel out, such as at time H, but also soon after D and some time after time K.

Exercise 9:

1x1 squares: $7\frac{1}{2}$

$\frac{1}{2} \times \frac{1}{2}$ squares: 7

Exercise 10:

13 quarter squares above: $3\frac{1}{4}$ units.

18 quarter squares below: $4\frac{1}{2}$ units.

Area between the curve and horizontal axis = $3\frac{1}{4} - 4\frac{1}{2} = -1\frac{1}{4}$.